



K23P 3113

Reg. No. :

Name :

I Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, October 2023
(2023 Admission)
MATHEMATICS
MSMAT01C03 : Real Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Let A be the set of all sequences whose elements are the digits 0 and 1. Show that the set A is uncountable.
2. Define a dense set. Give an example with justification.
3. Show that continuous image of a compact metric space is compact.
4. Let f be defined for all real x , and suppose that $|f(x) - f(y)| \leq (x - y)^2$ for all real x and y . Prove that f is constant.
5. If $f \in R(\alpha)$ on $[a, b]$, show that $|f| \in R(\alpha)$ and $|\int_a^b f d\alpha| \leq \int_a^b |f| d\alpha$.
6. Show that the curve $\gamma : [0, 2\pi] \rightarrow \mathbb{C}$ defined by $\gamma(t) = e^{it}$ is rectifiable.

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. For any finite collection G_1, G_2, \dots, G_n of open sets, show that $G_1 \cap G_2 \cap \dots \cap G_n$ is open. Give an example to show that finiteness in the above statement is essential.

P.T.O.



8. a) Show that every neighborhood is an open set.
- b) Discuss the continuity of the function $f(x) = \begin{cases} x + 2, & \text{if } -3 < x < -2. \\ -x - 2, & \text{if } -2 \leq x < 0. \\ x + 2, & \text{if } 0 \leq x < 1. \end{cases}$
9. Show that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y .
10. Suppose f is a continuous mapping of $[a, b]$ into \mathbb{R}^k and f is differentiable in (a, b) . Show that there exist $x \in (a, b)$ such that $|f(b) - f(a)| \leq (b - a)|f'(x)|$.
11. If f is continuous on $[a, b]$, show that $f \in R(\alpha)$.

PART - C

Answer **any three** questions. **Each** question carries **13** marks.

12. Show that every k -cell is compact.
13. Let f be a continuous mapping of a compact metric space X into a metric space Y . Show that f is uniformly continuous on X .
14. State and prove L'Hospital rule.
15. a) If γ' is continuous on $[a, b]$, then show that γ is rectifiable, and
$$\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt.$$
 b) If $f, g \in R(\alpha)$ on $[a, b]$, show that $f + g \in R(\alpha)$ on $[a, b]$.
16. a) If f is a real continuous function on $[a, b]$ which is differentiable in (a, b) , show that there is a point $x \in (a, b)$ at which $f(b) - f(a) = (b - a)f'(x)$. Is this result true for complex functions. Justify your answer.
 b) If P^* is a refinement of P , show that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.